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# NAVPAC MESA DATA PROCESSING – MATHEMATICAL DESCRIPTION

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The Navigation Package (NAVPAC) is a satellite-borne data collection system consisting of two sensor subsystems: a receiver/data subsystem capable of tracking up to three Navy Navigation Satellites (NAVSATs) simultaneously and recording Doppler, refraction, and timing information and a miniature electrostatic accelerometer (MESA) capable of measuring accelerations due to atmospheric drag, winds, and orbit adjust thrusts. This report provides a detailed mathematical description of the NAVPAC MESA data processing procedures that include preprocessing,

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low-pass filtering, and orbit determination. Also included is an overview of the hardware operation required for the correct interpretation of the MESA data. Additional Keywords: NAVPAC Computer program.

## FOREWORD

The work described in this report was done by the Space and Surface Systems Division of the Naval Surface Weapons Center (NSWC) in support of the Defense Mapping Agency's Navigation Package (NAVPAC) program. Technical contributions to this work were made by C. Branch, B. Hermann, A. Buonaguro, and J. Blanton. Programming support was provided by A. Fisher, L. Szeto, G. Barker, E. Colquitt, and T. Burgess.

This report has been reviewed and approved by C. W. Duke, Jr., Head, Space and Surface Systems Division.

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# INTRODUCTION

The Navigation Package (NAVPAC) is a satellite-borne data collection system developed for the Defense Mapping Agency (DMA). It consists of two sensor subsystems:

1. A receiver/data subsystem capable of tracking up to three Navy Navigation Satellites (NAVSATS) simultaneously and recording integrated Doppler, first-order ionospheric refraction, and timing information

2. A miniature electrostatic accelerometer (MESA) capable of measuring accelerations due to atmospheric drag, winds, and orbit adjust thrusts.

The receiver/data subsystem, which also provides the power, command, telemetry, and MESA data storage functions, was designed and built by the Applied Physics Laboratory of Johns Hopkins University (APL/JHU) according to specifications provided by the Naval Surface Weapons Center (NSWC). The MESA was designed and built by Bell Aerospace TEXTRON according to specifications provided by the Air Force Geophysics Laboratory (AFGL) and NSWC.

The NAVPAC satellite-to-satellite tracking (SST) Doppler data and the accelerometer data are used by NSWC for orbit determination for the host satellite. In the future, these data may also be used for gravity field model improvement. The MESA data are also used by AFGL for atmospheric density and wind studies.

NAVPAC stores the collected data in a core storage unit until it becomes full (approximately one rev). At that time, the data are automatically transferred to an onboard tape recorder. Daily, the data accumulated on this tape recorder are telemetered to a ground station, recorded on analog tape, and provided to NSWC. Here, the analog tape is digitized, preprocessed in the "Quick Look" mode using preliminary trajectories, and a preliminary data quality evaluation is done. Later, the data are grouped for a given timespan and preprocessed in the "Normal" mode using the precise NAVSAT trajectories. After preprocessing (and filtering for the MESA data), the data are combined with the station tracking data for the host satellite in NSWC's orbit computation program, CELEST, to produce the required ephemerides.

The purpose of this report is to provide a detailed mathematical description of the NAVPAC accelerometer data processing procedures from preprocessing to orbit determination, including an overview of the hardware operation required for the correct interpretation of the MESA data. A similar description for the NAVPAC satellite-to-satellite tracking data is given in Reference 1.

## MESA SUBSYSTEM OVERVIEW

### FUNCTIONAL DESCRIPTION

The accelerometer consists of a flanged hollow cylindrical proof mass and housing, a temperature monitor, and supporting power and signal conditioning electronics. The proof mass is electrostatically suspended in the instrument's X and Y directions (perpendicular to the cylinder axis) by eight electrodes resident on a cylindrical carrier extending through the proof mass and in the instrument's Z direction (perpendicular to the flange) by annular electrodes on the proof-mass housing itself. The relative motion between the proof mass and its housing/carrier is sensed in three orthogonal directions, and restoring forces are generated in three separate feedback constrainment loops to keep the proof mass centered. Each restoring voltage is proportional to the input acceleration in the corresponding direction. These voltage signals form the starting point for the acceleration measurement.

High frequency acceleration inputs (>100 Hz) are highly attenuated due to the small spacing between the proof mass and its housing/carrier and the presence of gaseous nitrogen. Outside the constrainment loops, low-pass filtering of the voltage signals is done to remove unwanted satellite vibrational accelerations and to reduce the effects of aliasing. Active filters were added to the instrument for this purpose. The composite response of all circuitry up to this point (still an analog signal) is given for the most sensitive scale (C) in Figure 1. This figure indicates four orders-of-magnitude attenuation by 6 Hz—the first resonant frequency of the host satellite structure. The slope of this curve is not quite as steep for the other two scales (B and A).

Each filtered constrainment voltage is then fed to a voltage-to-pulse-rate converter. The pulses are accumulated in an up/down counter (to account for the sign of the input accelerations) for approximately 2.045 NAVPAC seconds. This counter is then read out and reinitialized. Each count can be converted to an average acceleration over 2.045 sec by application of scale factors determined using the instrument calibration data collected in ground tests to be discussed later.

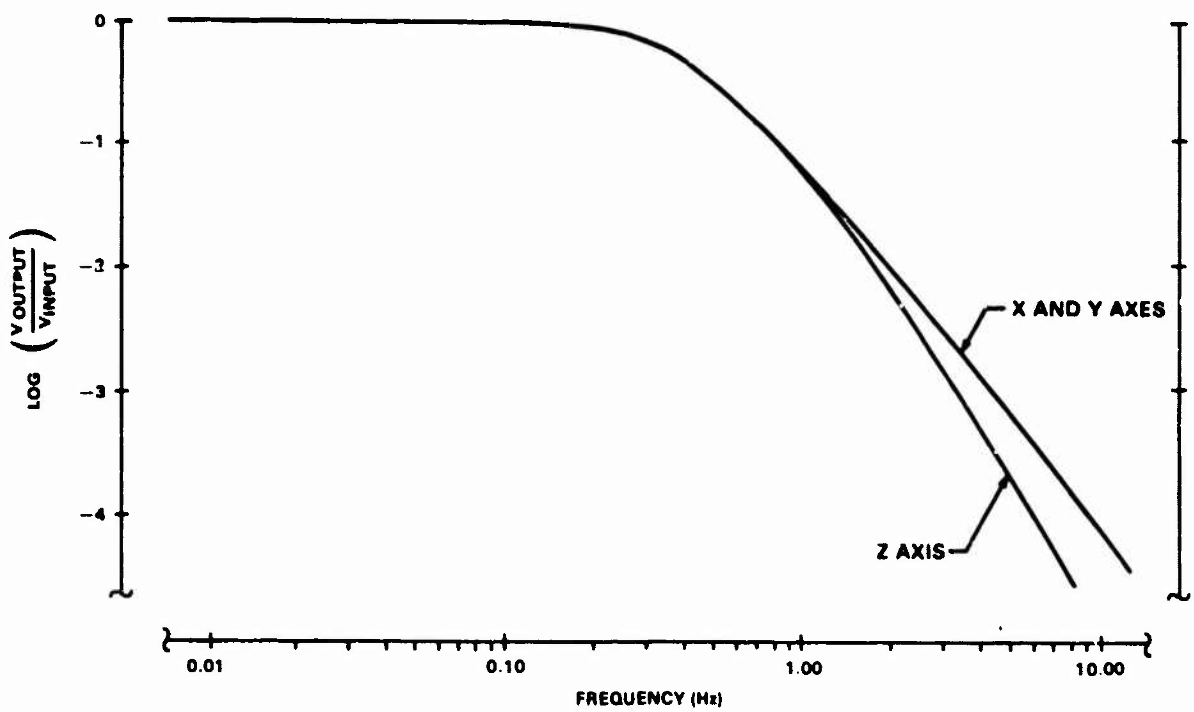


FIGURE 1. MESA C-SCALE COMPOSITE RESPONSE

Accelerations to be measured range from 0.012 g's for an orbit adjust thrust down to $10^{-8}$ g's for high-altitude drag. This wide measurement range is accommodated by hardware selected scaling. Any of three scales (A, B, or C) can be selected independently for each axis based on the pulse rate. If the maximum pulse rate is present for 0.25 sec on the Z axis or 2 sec on the X or Y axis, the instrument will uprange to the next less sensitive scale on the appropriate axis only. Downranging occurs when the pulse rate on a given axis is below a certain minimum value for the same time intervals specified above. These scale changes are asynchronous with the counting intervals so that a 2.045-sec sample containing a scale change on a given axis is bad for that axis only. The instrument operates on the C scale at all times other than during orbit adjust thrusts or other maneuvers. The resolution on a given scale depends on the maximum pulse rate, the counting interval, and the full-scale acceleration limit. A 15-KHz maximum pulse rate and a full-scale acceleration limit of 150 μg's for the 2.045-sec counting interval gives a resolution of approximately 0.005 μg's/pulse.

The instrument also contains a thermistor buried in the proof-mass housing very close to the proof mass to provide an accurate internal temperature. A voltage (0 to 5 v) proportional to temperature is provided continuously to the NAVPAC data subsystem.

Every 2.0447232 NAVPAC sec the contents of the up/down counters for each axis are read out into a 64-bit data word, as shown in Figure 2. For each axis 18 bits of data are present—15 bits containing the count, 1 bit for polarity information, and 2 bits for scale information. NAVPAC time (formatted as in Figure 3) is put into the data stream every 32 data words to provide timing information and a means to detect any missing data. The temperature monitor voltage is sampled once per telemetry frame—approximately every 114.5 sec. This voltage is digitized into an 8-bit word giving a resolution of about 0.25°F. The position of this MESA temperature word within a telemetry frame is given in Figure 4. Eight bits of telemetry information are put into the data stream every 2.0447232 sec also.

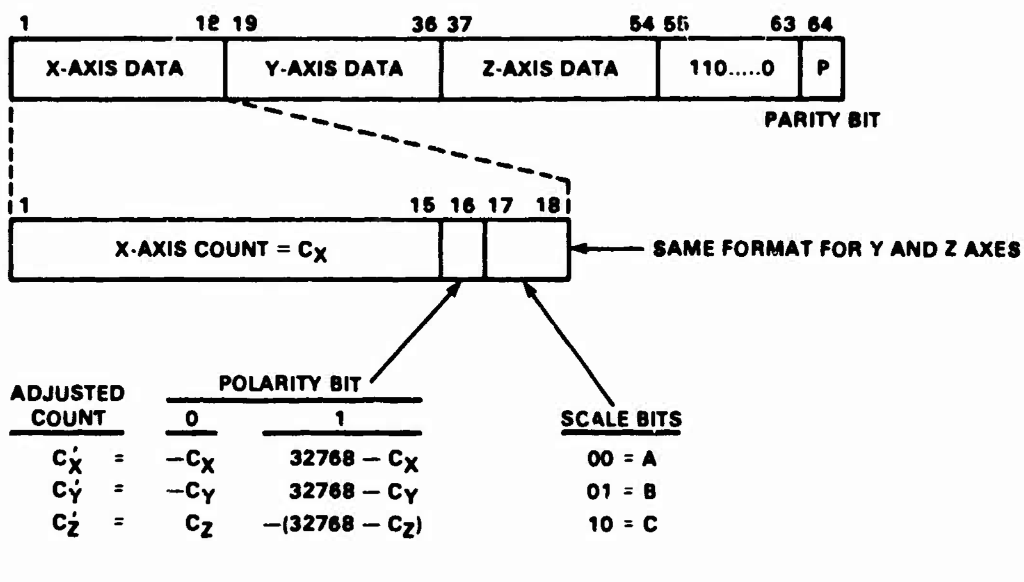


FIGURE 2. FORMAT OF THE 64-BIT ACCELEROMETER DATA WORD

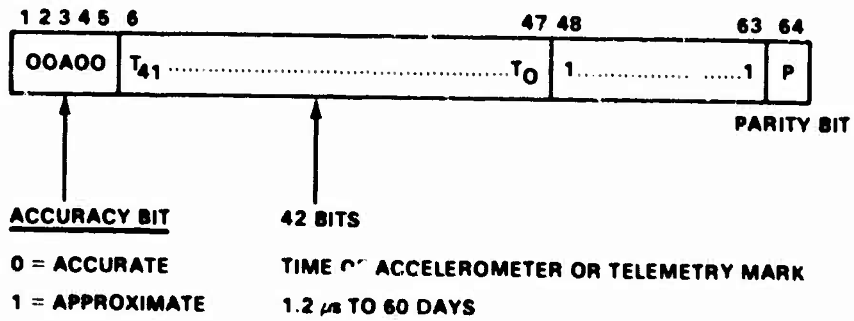


FIGURE 3. FORMAT OF THE 64-BIT TIME MARK WORD

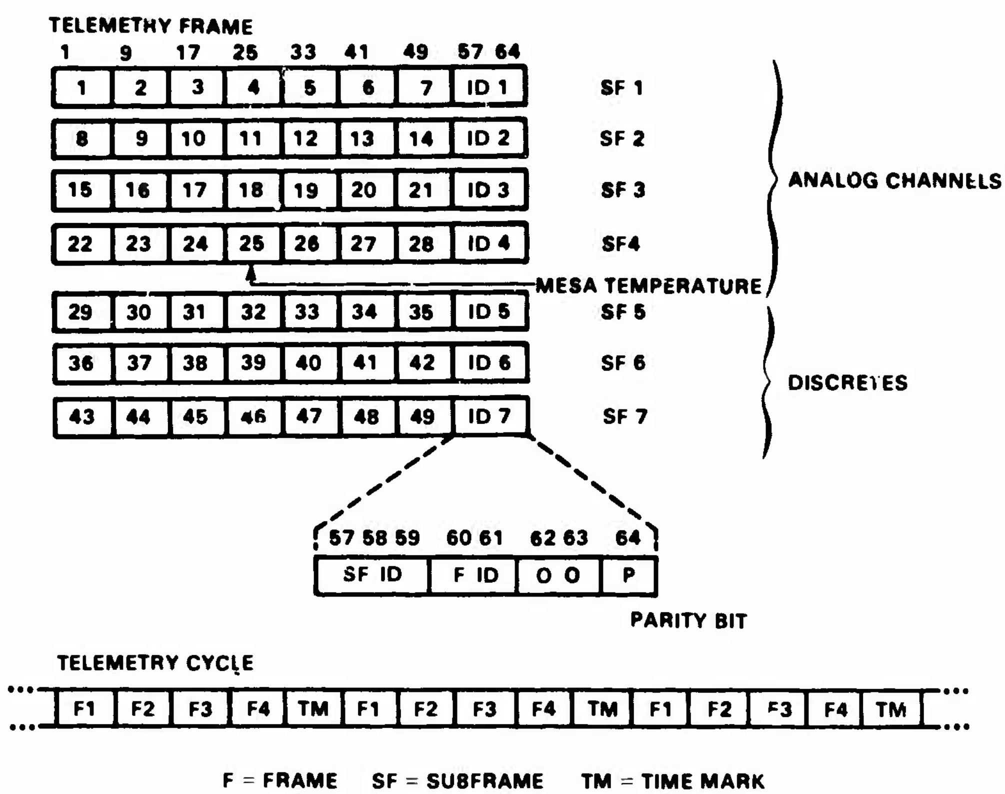


FIGURE 4. FORMAT OF THE TELEMETRY DATA BIT STREAM

Calibration data are collected in ground tests of the instrument to provide the scale factors required to convert from counts to μg's. In the ground tests, test equipment is used to provide the 1-g suspension force for the axis mounted vertically and then the instrument is tilted slightly to introduce a component of gravity in the desired axis. Calibration consists of collecting the output of the accelerometer for each axis on each scale for a known input acceleration at a given stabilized internal temperature. For a given axis, scale, and temperature the input acceleration is varied between plus and minus full scale to provide data for determining the linearity of the scale factor. Then the temperature is varied between its possible extremes to provide the scale factors as a function of temperature. All conversions between μg's and km/sec$^2$ are based on the magnitude of gravity at the instrument manufacturer's location (Bell Aerospace TEXTRON) of 980.388 cm/sec$^2$. In addition, calibration data for the thermistor is provided to relate voltage to temperature.

The above discussion was only intended to give the reader a sufficient understanding of the accelerometer subsystem and certain aspects of the data subsystem operation for correctly interpreting the NAVPAC MESA data. Reference 2 contains more detailed information on the mechanical and electrical characteristics of the MESA and the calibration technique. Reference 3 contains a detailed description of the NAVPAC data subsystem.

## ACCELERATIONS MEASURED

The NAVPAC accelerometer is rigidly mounted away from the center of mass (c.m.) of a three-axis stabilized host satellite with its Z axis parallel to the satellite's X axis (along-track axis), its X axis parallel to the satellite's Y axis (cross-track axis) and its Y axis parallel to the satellite's Z axis (radial axis). This is depicted in Figure 5. This alignment, of course, is not perfect. Alignment of the proof mass relative to the case, the case relative to the mounting fixture, and the mounting fixture relative to the attitude reference module on the satellite are measured before flight. However, during flight, uncertainties in this last alignment due to thermal deformations are present and the attitude of the satellite with respect to inertial space is uncertain due to attitude control system errors. Only constant known attitude errors are accounted for in the accelerometer data reduction.

Based on the definitions given in Figure 5, the instantaneous acceleration at the MESA location is given by:

$$\ddot{\bar{r}}_{MESA} = \ddot{\bar{r}} + \mu\left(\frac{\bar{r}_{MESA}}{|\bar{r}_{MESA}|^3} - \frac{\bar{r}}{|\bar{r}|^3}\right) + \bar{\omega} \times (\bar{\omega} \times \bar{R}) + \dot{\bar{\omega}} \times \bar{R} + \ddot{\bar{r}}_{Vib}.$$

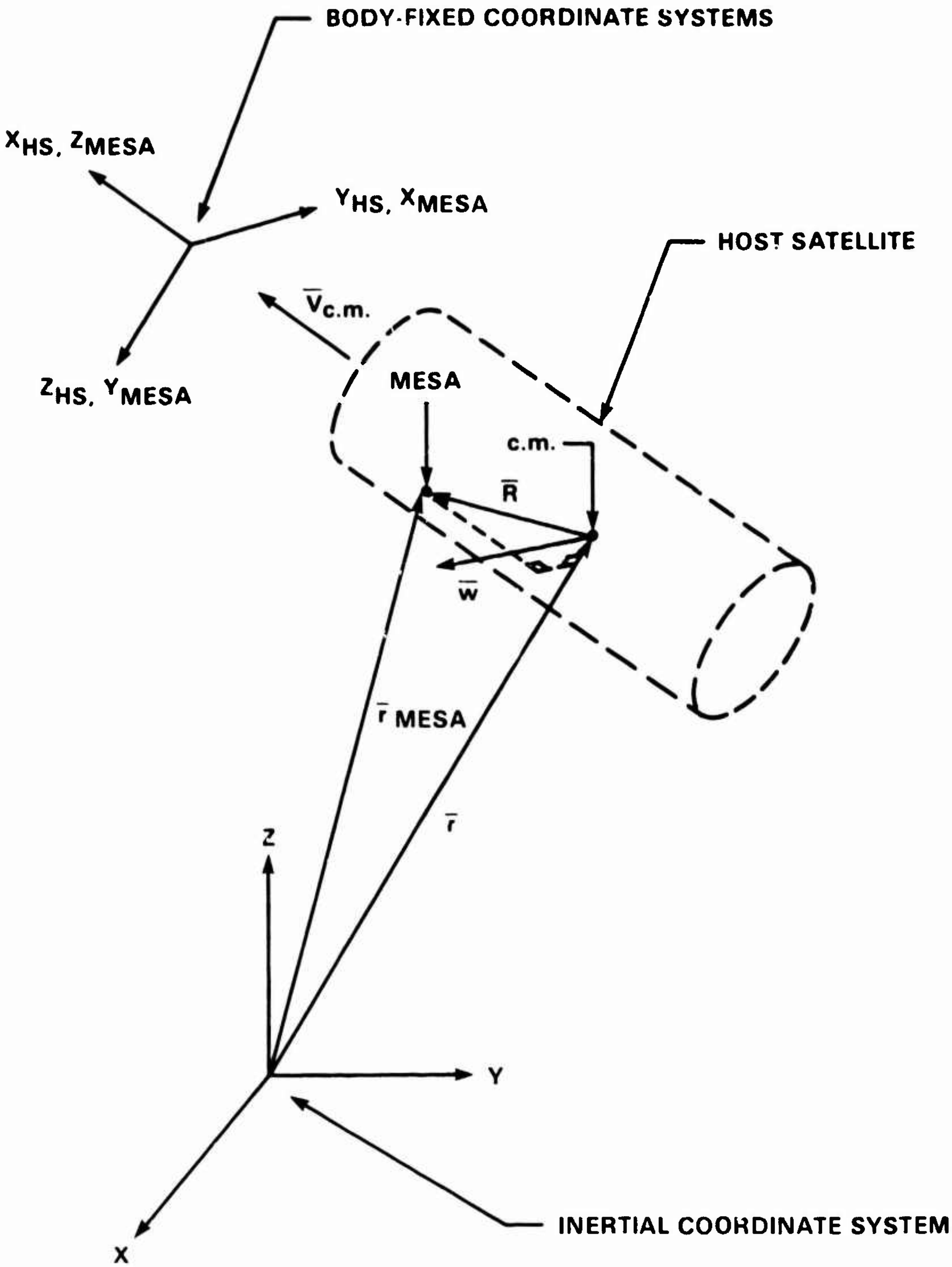


FIGURE 5. REFERENCE FRAME AND VECTOR DEFINITION

where

$\mu$ = GM = gravitational constant times the mass of the Earth and $\bar{\omega}$ = satellite's angular velocity vector.

The first term on the right is the translational acceleration of the c.m. of the host satellite due to all non-conservative forces such as atmospheric drag, solar radiation pressure, orbit adjust thrusts, and attitude control thrusts. This is the acceleration required for orbit determination. The second term is the gravity gradient acceleration (ignoring all but the central force term) due to the fact that the c.m. of the satellite is actually in free-fall and the accelerometer is constrained relative to the c.m. The third term is the centripetal acceleration caused by any rotation of the measurement frame. The primary source is the rotation required to maintain the desired attitude. Secondary sources include attitude maneuvers and torque-induced attitude rates within the deadbands of the control system. The fourth term is the transverse acceleration caused by changes in the angular velocity vector such as during attitude control thrusts or due to unbalanced torques acting on the satellite. The last term is the vibrational accelerations excited by the attitude thrusts and other satellite activity. These accelerations have frequencies as low as 0.1 Hz.

As mentioned above, the actual measurement output from the accelerometer is the average acceleration over a 2.045-sec interval based on a low-pass filtered instantaneous acceleration history. All of the accelerations discussed above except those due to attitude thrusts and vibrations are low frequency and would therefore be accurately measured. Since the attitude thrusts are of much shorter duration than the sample interval (<.05 seconds vs. 2.045 seconds) and produce instantaneous accelerations (even after filtering) above full scale on the C scale, a noise spike appears in the data. However, since the attitude thrusters fire in the satellite Y-Z plane, none of these spikes appear in the most important MESA Z-axis (along-track) direction.

With regards to the vibrational accelerations, the measurement process can be interpreted as instantaneous sampling of a continuous function consisting of 2.045-sec integrated accelerations. The effect of this integration is essentially to preserve the frequency content but to attenuate each component's magnitude by a factor of $\frac{\sin \pi f\Delta t}{\pi f\Delta t}$ where f = vibrational frequency in Hz and $\Delta t$ = 2.045 sec. This corresponds to a low-pass filter with a slope of 6 dB/octave. The sampling causes aliasing of frequencies above 0.25 Hz into the range from 0. to 0.25 Hz. This aliasing problem is the reason for the filtering done on the signal before the voltage-to-pulse-rate conversion. This is an idealized description of what happens and is complicated by the phase changes that occur in the vibrations during a measurement that contains one or more attitude thrusts. Some low-frequency vibrational accelerations are still present in the measurements.

In addition to the above accelerations, the electronics within the MESA also introduce biases and random noise into the measurements. The noise level is higher on the MESA X and Y axes than on the Z axis due to the curvature of the electrodes, larger spacing between the proof mass and the electrodes, and decreased resolution.

## DATA PREPROCESSING (NAVPAC)

The primary purpose of preprocessing the raw NAVPAC accelerometer data (using the NAVPAC computer program) is to convert it into a form usable by the available plotting programs and by the FOURCO low-pass filter program. This includes time tagging the data, converting the data to the appropriate units, applying known corrections to the data, and editing the data to ensure continuous records and no "bad" or wild points. In addition, time-tagged temperature and accelerometer raw data files are created for AFGL. The raw data are input as a stream of 64-bit words as defined in the previous section–all telemetry data words for a given span followed by all accelerometer data words. Additional input consists of the calibration coefficients for temperature and scale factor determination. Two modes of preprocessing are possible: "Quick Look" and "Normal." The MESA temperature processing is the same in each mode except that the times are referenced to different epochs. For the accelerometer data each mode is different as explained below. All timing for both the telemetry and accelerometer data is based on integer multiples of 2.0447232 NAVPAC seconds. The epoch to which all times are referenced in the preprocessing is the beginning of a specified day for the "Quick Look" mode and the host satellite long-arc trajectory epoch for the "Normal" mode.

### TEMPERATURE PROCESSING

The purpose of the temperature processing is to time tag and convert all MESA temperatures to °F for plotting, for determination of scale factors in the "Normal" MESA data preprocessing mode, and for use by AFGL. Each frame of telemetry data contains exactly one 8-bit MESA temperature value located in subframe 4, TLM channel 25 as indicated in Figure 4. To convert this 8-bit raw count, $C_T$, into a temperature monitor voltage, $T_v$, the following formula is used:

$$T_v \text{ (volts)} = (255 - C_T) \times 0.0196$$

The temperature monitor voltage is then converted to a temperature reading by the following formula:

$$T(^\circ F) = c_0 + c_1 T_v + c_2 T_v^2 + c_3 T_v^3 + c_4 T_v^4$$

where $c_n$, n = 0, 1, 2, 3, 4 are input calibration coefficients.

The time tag for each MESA temperature is based on $\tau_{TLMTM}$—the most recent NAVPAC telemetry time mark (in seconds) present in the data stream adjusted if necessary to account for any recycling of the NAVPAC clock (see Reference 1)—as follows:

$$t_T = (\tau_{TLMTM} - 8.1788928 + (\text{Frame \#} - 1) \times 114.5044992 + 57.2522496)_{cal}$$

where $t_T$ = Coordinated Universal Time (UTC) time of the MESA temperature reading

8.1788928 = number of NAVPAC seconds between the time at which channel 1 is sampled and $\tau_{TLMTM}$

114.5044992 = number of NAVPAC seconds required to collect one frame of telemetry data

57.2522496 = number of NAVPAC seconds between times for sampling channel 1 and channel 25

$(\ )_{cal}$ – This notation means substitution of the NAVPAC time within ( ) into the time calibration equation from Reference 1 repeated here as

$$t = t_{R_{recomputed}} + A + (1 + B)\tau + C\tau^2 \qquad (1)$$

where $t_{R_{recomputed}}$ = recomputed reset time (in UTC seconds)

$\tau$ = NAVPAC time (in seconds)

A,B,C = clock calibration coefficients

All resulting UTC times, $t_T$, are then referenced to the appropriate epoch—the beginning of a specified day in the "Quick Look" mode and the epoch of the host satellite long-arc trajectory in the "Normal" mode. MESA temperatures that occur before the first $\tau_{TLMTM}$ are correctly time tagged by subtracting integer multiples of 114.5044992 sec from the uncalibrated time for the first temperature after this time mark and calibrating the result. All time/temperature pairs are put on a file for plotting and the file from the "Normal" mode is required as input for the accelerometer "Normal" mode processing.

## "QUICK LOOK" PROCESSING

The primary purpose of the "Quick Look" processing is to provide a file of time-tagged measured accelerations in $\mu$g's for plotting purposes. Plots are the only means used to evaluate the data within a few days after data collection. In addition an accelerometer raw data file with calibrated time marks is created for AFGL. To do this processing the following items are required: clock calibration coefficients and scale factor calibration coefficients.

The 64-bit data words are processed sequentially one word at a time. Each NAVPAC MESA time mark, $\tau_{MESATM}$, is adjusted if necessary to account for any recycling of the NAVPAC clock and then calibrated using equation (1) above, i.e.,

$$t_{MESATM} = (\tau_{MESATM})_{cal}$$

This UTC time is then referenced to the beginning of the specified day. Each MESA time mark corresponds to the end of the first count present in the data stream immediately following the time mark. The time tag to be associated with this first count is defined to be the middle of the count interval since the measurement is actually the average acceleration over this interval, i.e.,

$$t_1 = t_{MESATM} - 1.0223616 \text{ sec}$$

Succeeding measurements have time tags

$$t_i = t_{i-1} + 2.0447232 \text{ sec} \qquad i = 2, 3, \ldots, 31, 32.$$

Only the time marks present are calibrated and then incremented by the NAVPAC time intervals instead of computing the NAVPAC time for each measurement and then calibrating the result.

To convert an accelerometer measurement from counts, polarity, and scale to $\mu$g's, the raw counts must first be adjusted based on the polarity bit to get the correct sign in the MESA coordinate system, as defined in Figure 5, as follows:

| Adjusted count | | Polarity bit 0 | Polarity bit 1 |
|---|---|---|---|
| $C'_X$ | = | $-C_X$ | $32768 - C_X$ |
| $C'_Y$ | = | $-C_Y$ | $32768 - C_Y$ |
| $C'_Z$ | = | $C_Z$ | $-(32768 - C_Z)$ |

Each adjusted count is then multiplied by an appropriate scale factor that is a function of axis and scale to convert it into $\mu$g's, i.e.,

$$a_{MESA} \text{ (in } \mu\text{g's)} = SF_{QL} \text{ (axis, scale)} \times C' \text{ for X, Y, and Z axes}$$

where $SF_{QL}$ units are $\mu$g's/count.

These scale factors are hardwired into the program and are derived based on a nominal MESA temperature and the calibration coefficients used in the "Normal" processing mode. Therefore the "Quick Look" mode output does not account for temperature variations. Also the accelerations are not corrected to the c.m. Both of these missing computations appear as biases or slowly varying, small magnitude perturbations which do not interfere with the basic purpose of the "Quick Look" processing–that of producing a plot for data evaluation within a few days after data collection.

Not every accelerometer data word is processed. Every $n^{th}$ measurement as specified by input is processed and printed except for a timespan specified by input or the last 360 frames (15 data words per frame) on the input file for which each measurement is processed, printed, and saved on the plot file. 360 frames correspond to approximately 10706 sec of continuous data. For all processed data words the raw counts, polarity bits, and scale bits are printed along with the associated time tag and the accelerations in $\mu$g's.

All MESA time marks are calibrated and printed. If a time mark is encountered before a complete set of 32 accelerometer data words is found, time tagging is restarted for the next accelerometer data word. If more than 32 data words occur before the next time mark is found, a time mark is created by adding 65.4311424 NAVPAC seconds to the previous time mark and time tagging is restarted. Groups of less than 32 data words and missing groups are filled in with flag words on the output raw data file only. Begin and end times for both the plot file and raw data file are provided at the end of the processing.

## "NORMAL" PROCESSING

The primary purpose of the "Normal" processing is to provide a file of time-tagged measured accelerations in km/sec$^2$ that have been adjusted for temperature effects, corrected to the c.m., edited, and made continuous for input into the FOURCO low-pass filter program. To do this processing the following items are required: clock calibration coefficients, the instrument's temperature history extracted from the telemetry data stream, scale factor calibration coefficients, a host satellite inertial trajectory, and a table of c.m. locations versus time. To ensure that the FOURCO program has enough data before and after the fit span in order to provide good filtered accelerations at the ends of the fit span, an additional 1400 sec of data is preprocessed before and after the fit span. The fit span is the span over which CELEST will fit a trajectory to the available data and is assumed identical to the long-arc trajectory integration span.

Again the 64-bit data words are processed sequentially one word at a time. The time tagging and conversion from raw to adjusted counts is done the same as in the "Quick Look" mode. Each adjusted count is then multiplied by an appropriate scale factor that is a function of axis, scale, and temperature to convert it to km/sec$^2$, i.e.,

$$a_{MESA} \text{ (in km/sec}^2\text{)} = SF_N \text{ (axis, scale, temperature)} \times C' \text{ for X, Y, and Z axes}$$

where $SF_N$ units are km/sec$^2$/count.

If $t_i$ is the measurement time tag, then linear interpolation in the temperature history table is used to get the MESA temperature, T, in °F at $t_i$, i.e.,

$$T(t_i) = T(t_j) + \left(\frac{T(t_{j+1}) - T(t_j)}{t_{j+1} - t_j}\right)(t_i - t_j)$$

where

$$t_j \leqslant t_i < t_{j+1} \qquad \{t_j\} = \text{set of all MESA temperature sampling times}$$

and

$$T = \text{MESA temperature in °F.}$$

Then the scale factor for a particular axis and scale is given by:

$$SF_N = (A + BT + CT^2 + DT^3)\left(\frac{g_{CAL} \times 10^{-6}}{2.0447232}\right)$$

where

$g_{CAL}$ = magnitude of gravity at the MESA calibration site = 0.00980388 km/sec²

A,B,C,D = input calibration coefficients for the particular axis and scale under consideration (some or all of B, C, and D may be 0.)

The first expression in parentheses on the right has units of μg's/pps. The factor $g_{CAL} \times 10^{-6}$ converts to km/sec²/pps, and the factor 2.0447232 converts to km/sec²/count. Scale factors are not recomputed for every accelerometer measurement but only every $n^{th}$ measurement as specified by input or whenever a scale change occurs on a particular axis or axes. This is done because the temperature variations have a period corresponding to the orbital period and a magnitude such that from measurement to measurement the change in temperature is extremely small (< 0.01 °F) and the resolution of the NAVPAC telemetry system for MESA temperature is much larger (0.25° F). Since scale changes are asynchronous with the count intervals, the first count for which a scale change is detected is flagged "bad" for that particular axis and processed as explained below in the discussion on editing. All scale changes on each axis are counted and summarized at the end of the processing.

As discussed in the Accelerations Measured section above, the location of the MESA at a position away from the c.m. of the host satellite results in three additional accelerations being measured–gravity gradient accelerations, centripetal accelerations, and transverse accelerations. The formulas required to compute these accelerations all involve the location of the accelerometer relative to the c.m. (ΔX, ΔY, ΔZ) in the MESA coordinate system, i.e.,

$$\Delta X(\text{in km}) = (X_{MESA} - X_{c.m.}) \times 2.54 \times 10^{-5}\ \text{km/in.}$$

and similarly for ΔY and ΔZ. The MESA coordinates in inches are hardwired into the program. The c.m. location at time $t_i$ must be computed by linear interpolation within the table of input c.m. locations versus time, i.e.,

$$X_{c.m.}(t_i) = X_{c.m.}(t_k) + \left(\frac{X_{c.m.}(t_{k+1}) - X_{c.m.}(t_k)}{t_{k+1} - t_k}\right)(t_i - t_k)$$

similarly for Y and Z

where

$t_k \leq t_i < t_{k+1}$. The times $\{t_k\}$ are actually input in days and seconds and must be converted to times from the fit span epoch for consistency with the accelerometer time tags.

The measured gravity gradient accelerations are approximated as follows in the MESA coordinate system:

$$a_{GG_X} = \frac{\mu \Delta X}{|\bar{r}|^3}$$

$$a_{GG_Y} = -\frac{2\mu \Delta Y}{|\bar{r}|^3}$$

$$a_{GG_Z} = \frac{\mu \Delta Z}{|\bar{r}|^3}$$

where

$\mu$ = GM = gravitational constant times the mass of the Earth (398600.8 $km^3/sec^2$)

$|\bar{r}|$ = distance from the center of the Earth to the c.m. of the satellite at the trajectory timeline nearest to the time required.

The measured centripetal accelerations are approximated as follows in the MESA coordinate system:

$$a_{C_X} = 0.$$

$$a_{C_Y} = -|\bar{\omega}_{ave.}|^2 \ \Delta Y$$

$$a_{C_Z} = -|\bar{\omega}_{ave.}|^2 \ \Delta Z$$

where

$|\bar{\omega}_{ave.}|$ = magnitude of the average angular velocity of the rotating measurement frame. This is hardwired into the program and assumes that the orbit is circular with a nominal semi-major axis.

Secondary sources of centripetal accelerations and variations from the above approximations are considered either negligible or are of such short duration that the low-pass filter would eliminate them. The total transverse accelerations are treated similarly. The measurements corrected to the c.m. are then given by

$$a_{c.m._X} = a_{MESA_X} - a_{GG_X} - a_{C_X} \qquad \text{similarly for the Y and Z axes.}$$

The gravity gradient and centripetal accelerations are actually computed every $m^{th}$ and $n^{th}$ measurement, respectively, as specified by input since all values required are slowly varying. At this point in the processing the measurements still are contaminated by instrument biases and noise, high frequency vibrational accelerations, and any residual accelerations not accounted for in the above correction procedure.

Extensive editing is done on the data to ensure that it is continuous and that all "bad" and wild points are replaced by realistic values. Ideally MESA time marks are separated by exactly 32 data words. If there are less than 32 data words present, it is assumed that the data present is continuous starting from the previous time mark and that the missing data is at the end of this interval. Linear extrapolation for each axis separately is used to fill in the missing data as follows:

$$a_{MESA}(t_r) = a_{MESA}(t_i) + \left(\frac{a_{MESA}(t_i) - a_{MESA}(t_{i'})}{t_i - t_{i'}}\right)(t_r - t_i)$$

for X, Y, and Z axes

where

$t_r$ = time tag for measurement to be added $> t_i$

$a_{MESA}(t_i)$ = last good measurement before missing data

$a_{MESA}(t_{i'})$ = last good measurement before $a_{MESA}(t_i)$

The "MESA" subscript is used here because this actually occurs before the measurements are corrected to the c.m. If more than 32 data words occur before a time mark, the next time mark expected is created based on the previous one. It is then assumed that the remaining data starts at this time and linear extrapolation is used to fill in all data until the next correct time mark is found. This may involve putting in several groups of 32 measurements. Messages are printed whenever a time mark is created and/or data is added.

As mentioned above in the discussion of the scale factor computations, measurements during which a scale change occurred on a particular axis are tagged "bad" for that axis. To replace this measurement succeeding measurements on this scale are examined to find two consecutive ones on the same scale. For all measurements starting at the "bad" measurement, linear interpolation is used to replace all intermediate values as follows:

$$a_{c.m.}(t_r) = a_{c.m.}(t_i) + \left( \frac{a_{c.m.}(t_i + R\Delta t) - a_{c.m.}(t_i)}{R\Delta t} \right) (t_r - t_i) \qquad r = 1, 2, \ldots, R-1$$

where

$t_i$ = time of last good measurement before scale change

$t_r = t_i + r\Delta t$ = time of replacement acceleration

$t_i + R\Delta t$ = time of first of two consecutive good points on same scale after $t_i$

$\Delta t$ = 2.0447232 sec

Wild point editing is done last. Each measurement is compared to the previous measurement for all axes. If

$$|a_{c.m.}(t_i) - a_{c.m.}(t_{i-1})| > 3. \times 10^{-8} \text{ km/sec}^2$$

for any axis, the mean and standard deviation of 4 points (2 on each side of $t_i$) are computed for each axis as follows:

$$\mu = [a_{c.m.}(t_{i-2}) + a_{c.m.}(t_{i-1}) + a_{c.m.}(t_{i+1}) + a_{c.m.}(t_{i+2})]/4.$$

$$\sigma = (\{[a^2_{c.m.}(t_{i-2}) + a^2_{c.m.}(t_{i-1}) + a^2_{c.m.}(t_{i+1}) + a^2_{c.m.}(t_{i+2})]/4.\} - \mu^2)^{1/2}$$

Then if $|a_{c.m.}(t_i) - \mu| > 5\,\sigma$ for any axis, set $a_{c.m.}(t_i) = \mu$ for each axis. The total number of measurements edited is then accumulated and summarized at the end of the processing.

All time marks are calibrated and printed but only every $n^{th}$ accelerometer measurement is printed. Information included are the raw counts, polarities, scales, time tag (in seconds from the fit span epoch), and acceleration components (in km/sec$^2$) given in the MESA

coordinate system. Whenever the scale factors are recomputed, the temperature, time, and current scale factors for each axis for the scale in use are printed. Whenever either the gravity gradient or centripetal accelerations are recomputed, the time and acceleration correction components are printed. The output file has the accelerations reordered to be given in the host satellite coordinate system, i.e., $a_{X_{HS}} = a_{Z_{MESA}}$, $a_{Y_{HS}} = a_{X_{MESA}}$, and $a_{Z_{HS}} = a_{Y_{MESA}}$.

## LOW-PASS FILTERING (FOURCO)

After the NAVPAC MESA data have been preprocessed, the data must be low-pass filtered to remove or attenuate instrument random noise, the remaining vibrational accelerations (either measured directly or due to aliasing), and other residual high frequency accelerations. Residual high frequency accelerations, even though they may be true translational accelerations, must be removed from the measurements because of the method chosen for orbit determination. This method (to be described in the next section) involves putting the filtered accelerations directly into a numerical integration algorithm. Therefore, the highest frequency contained in the input accelerations should be approximately an order of magnitude less than the inverse of the integration step size.

A non-recursive symmetric digital low-pass filter with optional coefficient weighting was selected because of its simplicity and its linear phase (constant time delay) characteristics. The filter is defined as follows:

Let $f_c$ = cutoff frequency (in Hz) $\qquad \omega_c = 2\pi f_c$

$f_s$ = sample frequency (in Hz) $= \dfrac{1}{\Delta t} \qquad \omega_s = 2\pi f_s$

$\Delta t$ = 2.0447232 sec

N = order of the filter (assumed even)

Then the recursion relationship that gives the filtered acceleration at time $t_i$ as a function of N previous, the current, and N future input measurement values (i.e., 2N + 1 points) is given by:

$$a_{filt.}(t_i) = c_0 \, a_{c.m.}(t_i) + \frac{1}{2} \sum_{n=1}^{N} c_n \left[a_{c.m.}(t_i - n\Delta t) + a_{c.m.}(t_i + n\Delta t)\right]$$

for X, Y, and Z axes

where

$$c_0 = \frac{2\omega_c}{\omega_s}$$

$$c_n = \frac{2}{n\pi} \sin\left(\frac{2n\pi\omega_c}{\omega_s}\right) \qquad n = 1, 2, \ldots, N$$

To reduce the Gibb's phenomena overshoot of the corresponding transfer function in the vicinity of the cutoff frequency, the coefficients can be multiplied by weights associated with the Hamming window given by

$$w_n = 0.54 + 0.46 \cos \frac{n\pi}{N} \qquad n = 0, 1, \ldots, N,$$

i.e., replace $c_n$ by $w_n c_n$ in the above formula.

All three axes are filtered concurrently. Filtered accelerations at each time $t_i$ are computed but not all are saved. Output accelerations are only required at the integration timelines $t_m$ as determined by CELEST (every 30 sec). The filtered acceleration with a time tag $t_i$ such that $|t_m - t_i|$ is a minimum is assigned to the time $t_m$. Since CELEST requires 10 timelines before and 8 timelines after the fit span, the filtered acceleration file starts 300 sec before and ends 240 sec after the fit span. The 1400 sec of extra data at each end of the preprocessor output file is required to ensure good filtered acceleration values at these times based on a value of $N \leq 512$.

## ORBIT DETERMINATION (CELEST)

### GENERAL

After the NAVPAC MESA data have been preprocessed and low-pass filtered, it is in the proper form for use in CELEST–NSWC's orbit computation program. CELEST employs a classical weighted least-squares differential correction technique to fit satellite initial conditions and force and measurement model parameters to various types of observations. The program consists of four major sections:

1. ORBGEN–generates the reference trajectories and dynamic partial derivatives

2. FILTER–edits and determines weights for the data and forms pass matrices

3. BSOLVR–expands and combines pass matrices to obtain solutions and computes diagnostics

4. COVAR–propagates the solutions and their covariances to produce the fitted trajectory

CELEST is unlike most orbit computation programs in that it employs the "pass matrix" concept. Pass matrices are essentially normal equations based on data from each pass separately. The FILTER section forms the pass matrices as an integral part of its data editing and weight determination procedure. In the BSOLVR section, the solution for a given span (either a long or a short arc) is based on pass matrices with times-of-closest-approach in this span only. These so-called canonical pass matrices are updated to the epoch of the fit span and then certain expansions have to be performed to adjust the equations to reflect the actual drag and thrust profiles. Reference 4 contains a complete mathematical description of the CELEST program before it was modified to handle the NAVPAC-related data processing procedures. Reference 1 describes the modifications and additions to CELEST that were specifically designed to refine the processing procedures for station tracking data for the host satellite and/or the NAVSATs (single-satellite mode) and to make possible processing of the NAVPAC SST data (multisatellite mode).

The filtered accelerations are included in the orbit computation procedures by replacing the drag, radiation pressure, and orbit adjust thrust accelerations normally computed from models with these accelerations in the integration of the equations of motion for the host satellite. Variational equations are also generated and integrated so as to allow improvement of parameters associated with the accelerometer data, namely scale factors and biases. By assuming separate biases but a common scale factor for the three axes, a one-to-one correspondence between scale factor and drag coefficient and between bias and continuous thrust exists in the program, i.e., the scale factors replace drag coefficients and biases replace thrusts. This correspondence resulted in only the ORBGEN section having to be modified to handle the accelerometer data. The other three sections remained unchanged. This also implies that the scale factors can be segmented in both the long-arc and short-arc fits, i.e., different scale factors for specified subintervals of the fit span can be solved for. However, each scale factor must have the same a priori uncertainty. The changes made to the ORBGEN section are given below.

## INTEGRATION (ORBGEN)

At each trajectory integration timeline, $t_m$, the filtered acceleration, $\bar{a}_{filt.}$, has to be transformed from the MESA proof-mass axes to the host satellite coordinate system to the earth-centered inertial coordinate system being used for the equations of motion. This transformation is represented by the matrix product

$R_{HS \to I} T$

where

$R_{HS \to I} = (\hat{U}_1 \; \hat{U}_2 \; \hat{U}_3)$ = transformation matrix required to convert from the host satellite coordinate system to the inertial system

$$\hat{U}_1 = \frac{\bar{r} \times (\dot{\bar{r}} \times \bar{r})}{|\bar{r} \times (\dot{\bar{r}} \times \bar{r})|}$$

$$\hat{U}_2 = \frac{\dot{\bar{r}} \times \bar{r}}{|\dot{\bar{r}} \times \bar{r}|}$$

$$\hat{U}_3 = -\frac{\bar{r}}{|\bar{r}|}$$

$\bar{r}, \dot{\bar{r}}$ = inertial position and velocity of the host satellite at $t_m$

$$T = \begin{pmatrix} 1 & -\alpha & \beta \\ \alpha & 1 & -\gamma \\ -\beta & \gamma & 1 \end{pmatrix}$$ = constant transformation matrix used to account for any misalignment between the proof-mass axes and the host satellite coordinate system axes

$\alpha$, $\beta$, and $\gamma$ are small angles (in radians) about the host satellite's Z, Y, and X axes, respectively.

The $R_{HS \to I}$ transformation is an approximation to the actual case because the $\hat{U}_3$ direction should be perpendicular to the Earth's ellipsoid and not through the center of the Earth. The maximum error in this approximation is less than 0.2°.

The equations of motion for the host satellite are then given by

$$\bar{a} = \bar{a}_{Conservative} + R_{HS \to I} T (SF \, \bar{a}_{filt.} - \bar{B})$$

where

$\bar{a}$ = total acceleration in the inertial system

$\bar{a}_{\mathrm{Conservative}}$ = sum of the accelerations due to gravity of the Earth, Sun, and Moon and accelerations due to the tidal distortion of the Earth

SF = scale factor parameter with initial value input (nominally 1.0)

$\bar{B}$ = bias parameters with initial values input in km/sec².

Since SF and $\bar{B}$ are to be parameters of solution, variational equations must be generated and integrated for these quantities. The equations are given by

$$\ddot{\bar{h}}_1 = \underset{3\times 3}{\frac{\partial \bar{a}}{\partial \bar{r}}}\,\bar{h}_1 + \underset{3\times 3}{\frac{\partial \bar{a}}{\partial \dot{\bar{r}}}}\,\dot{\bar{h}}_1 + \underset{3\times 1}{\frac{\partial \bar{a}}{\partial SF}} \qquad \text{where } \underset{3\times 1}{\bar{h}_1} = \frac{\partial \bar{r}}{\partial SF}$$

$$\ddot{\bar{h}}_{2_i} = \underset{3\times 3}{\frac{\partial \bar{a}}{\partial \bar{r}}}\,\bar{h}_{2_i} + \underset{3\times 3}{\frac{\partial \bar{a}}{\partial \dot{\bar{r}}}}\,\dot{\bar{h}}_{2_i} + \underset{3\times 1}{\frac{\partial \bar{a}}{\partial B_i}} \qquad \text{where } \underset{3\times 1}{\bar{h}_{2_i}} = \frac{\partial \bar{r}}{\partial B_i} \qquad i = X,\ Y, \text{ and } Z$$

The non-homogeneous terms are given by

$$\frac{\partial \bar{a}}{\partial SF} = R_{HS\rightarrow 1}\,T\,\bar{a}_{filt.}$$

and

$$\frac{\partial \bar{a}}{\partial \bar{B}} = -\,R_{HS\rightarrow 1}\,T, \text{ i.e., } \frac{\partial \bar{a}}{\partial B_i} = i^{th} \text{ column of } -\,R_{HS\rightarrow 1}\,T \qquad i = X,\ Y, \text{ and } Z$$

The other partial derivatives of $\bar{a}$ are given by

$$\underset{3\times 1}{\frac{\partial \bar{a}}{\partial p}} = \underset{3\times 3}{\frac{\partial R_{HS\rightarrow 1}}{\partial p}}\,\underset{3\times 3}{T}\,(SF\,\underset{3\times 1}{\bar{a}_{filt.}} - \bar{B}) \qquad \text{for } p = x,\ y,\ z,\ \dot{x},\ \dot{y}, \text{ and } \dot{z}$$

where

$\frac{\partial R_{HS \rightarrow I}}{\partial p}$ means a matrix consisting of the partial derivative of each element of $R_{HS \rightarrow I}$ with respect to p

The matrix of partial derivatives $\frac{\partial R_{HS \rightarrow I}}{\partial p}$ is approximated by

$$\frac{\partial R'}{\partial p} \begin{pmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix} \quad \text{for } p = x,\ y,\ z,\ \dot{x},\ \dot{y}, \text{ and } \dot{z}, \text{ where}$$

$$R' \doteq \begin{pmatrix} \frac{\bar{r}}{|\bar{r}|} & \frac{\dot{\bar{r}}}{|\dot{\bar{r}}|} & \frac{(\bar{r} \times \dot{\bar{r}})}{|(\bar{r} \times \dot{\bar{r}})|} \end{pmatrix}$$

and $\frac{\partial R'}{\partial p}$ is formed in the "body-axes" thrust model in CELEST (See Reference 4). The R′ component vectors do not in general form an orthonormal set as do the $R_{HS \rightarrow I}$ component vectors. However, for a nearly circular orbit no significant error results by using the permuted and properly signed R′ transformation partial derivatives in place of the $R_{HS \rightarrow I}$ partial derivatives.

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